

# International Financial Markets

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# Outline

- Securities and International Financial Holdings
- The Mean Variance Portfolio Model
- Taking the model to the data

## Securities and International Financial Holdings

# Securities

- Securities are tradable assets of any kind.
  - debt securities (e.g., bonds)
  - equity securities (e.g., common stocks)
  - derivative contracts (e.g., forwards, futures, options, swaps)
- We will examine bonds and stocks: assets with safe and risky return respectively.
- To a first order, two are the moments that characterize a security:
  - Mean of its return
  - Variance of its return

# Motivation

Data for the fraction of domestic equity in overall equity holdings:

US	UK	Japan
.96	.82	.98

- - Is this behavior optimal?
- - Should investor hold more or less foreign equity?

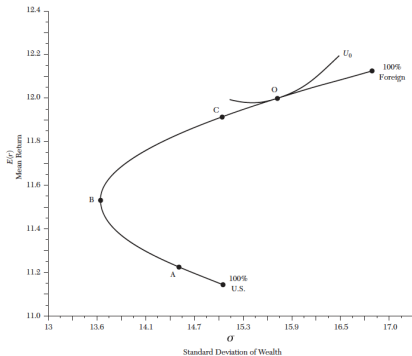
⇒ We need to figure out whether it is worth holding foreign assets.

# Is it Worth Holding Foreign Assets?

⇒ The following graph indicates that "Yes, it is".

- Note: A: 8% foreign portfolio. B: minimum variance portfolio. Compare C to A. We will go back to O.

**Figure:** Mean and Variance of a Portfolio of US S&P 500 & foreign EAFE fund (Morgan Stanley index)



# Foreign Assets Holdings

- What are the reasons that there is trade in assets?

In the previous example, we saw that it makes sense for US investors to hold foreign equities because they can get higher return with lower variance.

- From the foreign investors' point of view, it does not make sense unless they want safer returns.
- Still, would domestic agents hold foreign assets if they had to exchange return for variance?
- The Mean Variance Portfolio Analysis also popularized as the CAPM (Capital Asset Pricing Model) model gives us reasons to hold multiple assets if their returns are sufficiently uncorrelated.

## The Capital Asset Pricing Model (CAPM)



# Why Is There Trade in Assets: The CAPM Model

We will consider the CAPM model for assets of 2 countries.

Assume 2 assets:  $h$  (home, return  $R_h$ ) &  $f$  (foreign, return  $R_f$ ) both in levels

- Investor with wealth  $W$  chooses to invest a share  $\omega$  in one asset and  $1 - \omega$  in the other asset: overall return

$$R_p = \omega R_h + (1 - \omega) R_f$$

where we assume  $\omega \in [0, 1]$ , i.e. we do not allow the investor to short.

- His utility from holding this portfolio depends on the expected return  $E(R_p)$  and the variance of the return  $V(R_p)$  of this portfolio.

## Portfolio Returns

Recall:  $R_p = \omega R_h + (1 - \omega) R_f$ .

- Let  $\sigma_i^2 = E(R_i^2) - [E(R_i)]^2$  be the variance of the return of each portfolio  $i$  where  $i = h, f$ .

Let  $\rho_{hf} = \frac{\text{cov}(R_h, R_f)}{\sigma_h \sigma_f} = \frac{E[(R_h - ER_h)(R_f - ER_f)]}{\sigma_h \sigma_f}$  be the correlation of the returns.

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What is the expected return and variance of the overall portfolio?

- Expected return of portfolio:  $E(R_p) = \omega E(R_h) + (1 - \omega) E(R_f)$

# Portfolio Returns

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- Variance of the Return:

$$\begin{aligned} V(R_p) &= E(R_p - E(R_p))^2 = E(R_p)^2 - [E(R_p)]^2 \\ &= \omega^2 \sigma_h^2 + (1 - \omega)^2 \sigma_f^2 + 2\omega(1 - \omega) \rho_{hf} \sigma_h \sigma_f \end{aligned}$$

- Where we used the formula

$$\text{Var}(\omega X + (1 - \omega) Y) = \omega^2 \text{Var}(X) + (1 - \omega)^2 \text{Var}(Y) + 2\omega(1 - \omega) \text{cov}(X, Y)$$

## Preferences

Investor maximizes utility  $U(E(R_p), V(R_p))$  where  $U_1 > 0$ ,  $U_2 < 0$  by picking a share  $\omega$  of domestic assets (& thus,  $1 - \omega$  of foreign assets) in his portfolio.

- Investor's problem is

$$\max_{\omega \in [0,1]} U(E(R_p), V(R_p))$$

# Preferences

Investor maximizes utility  $U(E(R_p), V(R_p))$  where  $U_1 > 0$ ,  $U_2 < 0$  by picking a share  $\omega$  of domestic assets (& thus,  $1 - \omega$  of foreign assets) in his portfolio.

- What is the role of the preferences?

- Utility increases in the return of wealth and decreases in its variance.
- The substitution between return and risk determines the relative risk aversion,  $\gamma$ , where

$$\gamma \equiv -\frac{2WU_2}{U_1} > 0$$

## Preferences and Portfolio Choice

Consumer maximizes utility  $U(E(R_p), V(R_p))$  where  $U_1 > 0$ ,  $U_2 < 0$  by picking a share  $\omega$  of domestic assets (& thus,  $1 - \omega$  of foreign assets) in his portfolio.

- The consumer maximizes his utility by choosing  $\omega$  such that  $\frac{\partial U}{\partial \omega} = 0$ .  
Therefore,

$$U_1 \frac{\partial E(R_p)}{\partial \omega} + U_2 \frac{\partial V(R_p)}{\partial \omega} = 0$$

## Preferences and Portfolio Choice

Consumer maximizes utility  $U(E(R_p), V(R_p))$  where  $U_1 > 0$ ,  $U_2 < 0$ .

► The consumer maximizes his utility by choosing  $\omega$  such that  $\frac{\partial U}{\partial \omega} = 0$ .

$$U_1 (ER_h - ER_f) + U_2 \left[ \begin{array}{l} 2\omega\sigma_h^2 - 2(1-\omega)\sigma_f^2 + \\ + 2(1-\omega)\rho_{hf}\sigma_h\sigma_f - 2\omega\rho_{hf}\sigma_h\sigma_f \end{array} \right] = 0$$

$$\frac{U_1}{U_2} (ER_h - ER_f) = -2\omega\sigma_h^2 + 2(1-\omega)\sigma_f^2 - 2(1-\omega)\rho_{hf}\sigma_h\sigma_f + 2\omega\rho_{hf}\sigma_h\sigma_f$$

$$-\frac{U_1}{2U_2} (ER_h - ER_f) + \sigma_f^2 - \rho_{hf}\sigma_h\sigma_f = \omega [\sigma_h^2 + \sigma_f^2 - 2\rho_{hf}\sigma_h\sigma_f]$$



## Portfolio Diversification

Consumer maximizes utility  $U(E(R_p), V(R_p))$  where  $U_1 > 0$ ,  $U_2 < 0$ .

► The consumer maximizes his utility by choosing  $\omega$  such that  $\frac{\partial U}{\partial \omega} = 0$ .

$$\omega = \underbrace{-\frac{U_1}{2U_2} \frac{ER_h - ER_f}{\text{Var}(R_h - R_f)}}_{\text{higher potential returns from foreign stock}} + \underbrace{\frac{\sigma_f^2 - \rho\sigma_h\sigma_f}{\text{Var}(R_h - R_f)}}_{\text{minimum variance portfolio shares}}$$

$$= -\frac{U_1}{2U_2} \frac{W(Er_h - Er_f)}{W^2 \text{Var}(r_h - r_f)} + \frac{W^2 \text{Var}(r_f) - W^2 \text{Cov}(r_h, r_f)}{W^2 \text{Var}(r_h - r_f)}$$

where  $\text{Cov}(R_h, R_f) = \rho\sigma_h\sigma_f$  and  $\text{Var}(R_h - R_f) = \sigma_h^2 + \sigma_f^2 - 2\rho_{hf}\sigma_h\sigma_f$  and we define  $r_i = R_i/W$  for  $i = h$  and  $f$ .

- Notice: the lower the risk aversion, the higher weight put on the first term

Taking the model to the data

# A Look at the Data

- Lets look at the predictions of our simple model (Lewis 1999).

**Figure:** Cross-Country Returns and Optimal Foreign Portfolio under the CAPM model (Lewis 1999)

HOME BIAS IN INTERNATIONAL EQUITY MARKETS								
A. Summary Statistics for Returns								
	US	Canada	France	Germany	Italy	Japan	UK	EAFE
1. Means <sup>a</sup>	11.14	9.59	11.63	11.32	5.81	14.03	12.62	12.12
2. Standard Deviation <sup>b</sup>	15.07	18.66	23.33	20.28	26.18	22.50	23.97	16.85
3. Correlation Matrix:								
US	1.00	0.70	0.44	0.36	0.22	0.26	0.51	0.48
Canada	—	1.00	0.43	0.31	0.29	0.27	0.52	0.49
France	—	—	1.00	0.60	0.42	0.39	0.54	0.65
Germany	—	—	—	1.00	0.37	0.37	0.43	0.62
Italy	—	—	—	—	1.00	0.38	0.35	0.51
Japan	—	—	—	—	—	1.00	0.36	0.86
UK	—	—	—	—	—	—	1.00	0.71
EAFE	—	—	—	—	—	—	—	1.00
B. Foreign Portfolio Shares in Percent of Wealth								
	Actual	For $\gamma =$				Minimum Variance		
$\chi^f$	8.00	1	2	3	10	39.5		
		75.9	57.7	51.6	43.1			

Notes: Data in Panel A are monthly dollar indexes including reinvested dividends from Morgan Stanley. Panel B shares are calculated from the summary statistics in Panel A together with a relative risk aversion parameter ( $\gamma$ ) using equation (3) in the text.

<sup>a</sup> Annualized Mean Dollar Monthly Returns.

<sup>b</sup> Annualized Standard Deviation of Monthly Returns.

## Applying the Formula

Now, let us be really serious!

- What is the level of portfolio diversification that the theory implies?

- Choose the portfolio of US and EAFE equities. Lewis 1999 reports the following moments of the returns:
  - $Er_h = 11.14\%$ ,  $Er_f = 12.12\%$
  - $Cov(r_h - r_f) = 0.48 * 0.1507 * 0.1685 = 0.012$
  - $Var(r_h - r_f) = .1507^2 + .169^2 - 2 * 0.48 * .151 * .169 = 0.02673$
  - Choose a value for  $\gamma \equiv -\frac{2WU_2}{U_1}$ .
- Apply the CAPM formula with  $\gamma = 1$ .

$$\begin{aligned}\omega &= -\frac{U_1}{2WU_2} \frac{(Er_h - Er_f)}{Var(r_h - r_f)} + \frac{Var(r_f) - Cov(r_h, r_f)}{Var(r_h - r_f)} \\ &= \frac{1}{\gamma} \frac{0.111 - 0.121}{0.02673} + \frac{0.0227 - 0.01218}{0.02673} \simeq 24\%\end{aligned}$$

# Model and the Data

- Simple model would imply large diversification.

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# Motivation

⇒ **Diversification implies that investors should hold many foreign assets.**

**Some empirical work trying to address this puzzle:**

- Home bias in bond holdings is smaller than equities (>20 of public debt is held by nonresidents for the G7)
- Foreign direct investment holdings around 6-13 GNP for US, CAN, Germany, Japan
- Small step towards resolving the puzzle.
- A topic of vivid research (see, for example, Heathcote and Perri, Journal of Political Economy, forthcoming, The International Diversification Puzzle is not as Bad as you Think.)